JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
M.Tech I - Semester Examinations March/April-2011
ADVANCED FINITE ELEMENT ANALYSIS
(MACHINE DESIGN)

Time: 3hours   Max.Marks:60

Answer any five questions
All questions carry equal marks
- - -

1.   For the following differential equations and associated boundary conditions construct the variational statements, i.e., weak forms and wherever possible, quadratic functionals:

a)   One dimensional heat conduction/convection.

$$\frac{-d}{dx}\left(a\frac{du}{dx} \right) + f = 0 \quad \text{for } 0 < x < 1.$$

$$u(0) = 0, \ a\frac{du}{dx} + h(u - u_\infty) = q \text{ at } x = 1$$

Where a and f are functions of x, and h, u_∞ and q are constants.

b)   Beam on elastic foundation

$$\frac{d^2}{dx^2}\left(b\frac{d^2 w}{dx^2} \right) + kw + f = 0 \quad \text{for } 0 < x < L$$

$$w = b\frac{d^2 w}{dx^2} = 0 \text{ at } x = 0, L$$

where b and f are functions of x, and k is a constant.   [12]

2.   (a) Discuss in detail about convergence requirements with an examples.
      (b) Explain weighted residual method discussing the one dimensional equation of heat conduction.   [12]

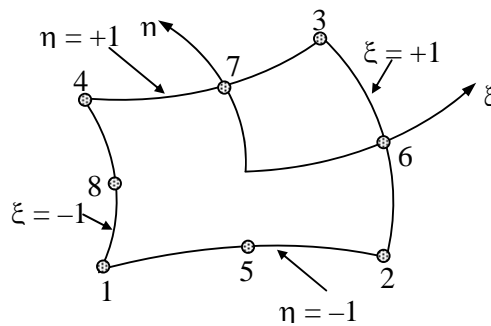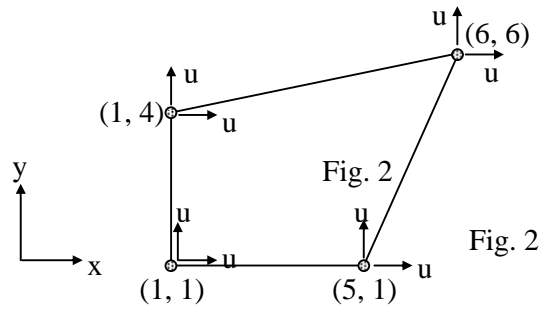3.   a)   Derive the interpolation functions at all nodes for the quadratic serendipity element, shown in Fig.1.

Fig. 1

b)   Using a 2×2 rule, evaluate the integral $\iint\limits_{\Omega}(x^2 + xy^2)\,dx\,dy$ by Gaussian quadrature where Ω denotes the region shown in Fig.2.

Fig. 2

4. a) Explain derivation of hybrid stress element.
   b) Explain the transformation procedure for transforming the element matrices from local coordinates to system coordinates by considering a suitable example.

5. (a) Derive the stiffness matrix [K] and the load vector for the two dimensional six nodded triangular element, also determine nodal displacements of triangular element, strain and stress of an element.
   (b) Determine the element equations for the plane stress element shown in Fig.3. The element has a 20 N/cm^2 load acting perpendicular to side jk and is subjected to a 15°C temperature rise.
   Thickness of the element = 2 cm; $E = 6 \times 10^6$ N/cm^2
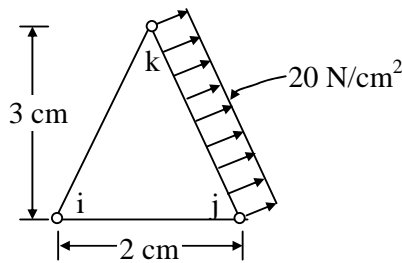 $\alpha = 7 \times 10^{-6}$ cm/cm °C; $\mu = 0.25$

Fig. 3

6. a) How a shell element can be formed? Explain.
   b) Compute the element matrices for the above shell element thus chosen, with a suitable example.

7. (a) Distinguish between consistent mass matrix and Lumped mass matrix..
   (b) Consider axial vibration of the steel bar shown in Fig.4, (i) develop the global stiffness and mass matrices and (ii) determine the natural frequencies and mode shapes using the characteristic polynomial technique.
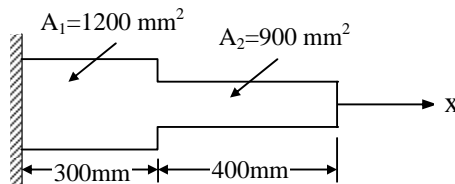

Fig. 4

8. (a) Explain the procedure with relevant equations for deriving the stiffness matrix for a plane bending elements using Mindlin Theory.
   (b) Explain the procedure with relevant equations for deriving the stiffness matrix for a plane bending elements using Kirchoff Theory. Also discuss about C_1 Continuity.

*****